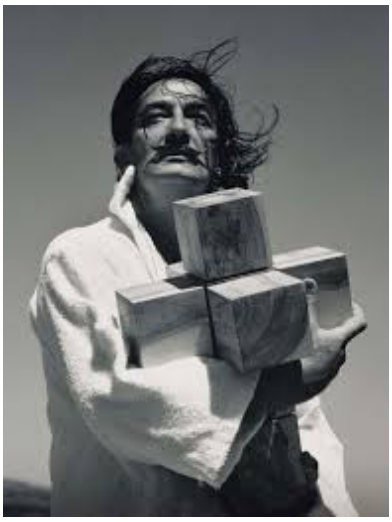
Dalí Museum, Saint Petersburg, Florida

Integrated Curriculum Tour Form

Education Department, 2014

TITLE:

"Salvador Dalí: High School Dalínian Mathematics"



SUBJECT AREA:

(VISUAL ART, LANGUAGE ARTS, SCIENCE, MATHEMATICS, SOCIAL STUDIES)

Visual Art, Mathematics (Next Generation Sunshine State Standards listed at the end of this document)

GRADE LEVEL(S):

Grades 9-12 and General Audience

DURATION: (NUMBER OF SESSIONS, LENGTH OF SESSION)

One session (45 minutes to one hour)

Resources: (Books, Links, Films and Information)

Books:

- *The Dalí Museum Collection: Oil Paintings, Objects and Works on Paper.*
- *The Dalí Museum: Museum Guide.*
- *The Dalí Museum: Building + Gardens Guide.*

Art and Mathematics Links:

- **Florida Art Education Association: www.faea.org**

- **National Art Education Association:** **www.arteducators.org**
- **National Core Art Standards:** **www.nationalartstandards.org**

- www.thedali.org
- http://www.artlex.com/ArtLex/m/mathematics.html
- http://plus.maths.org/issue3/fibonacci/index.html
- http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibinArt.html
- http://mathworld.wolfram.com/Golden Ratio.html
- http://educ.queensu.ca/~fmc/october2001/GoldenArt.htmml
- http://www-history.mcs.st-and.ac.uk/history/HistTopics/Art.html
- http://mathforum.org/library/topics/art/

Films:

- Dalí Condensed: 5 lecture series, Peter Tush, Curator of Education, Dali Museum You Tube Site.
- *Dimension Dalí*.
- Disney's *Donald in Mathematics Land:* Animated film about Greek proportions.

Information:

- Salvador Felipe Jacinto Dalí.
- May 11, 1904 – January 23, 1989.
- Figueres, Spain.

Pi, Irrational Number and Phi:

- Pi: The relation of the circumference to the diameter of every circle is the same. This constant is known as pi from the Greek perimetros and is expressed as approximately 3.14 or as 22/7. Pi is essential to descriptions of motion and mechanics and it is useful to everyday life.
- www.teachpi.org
- Irrational Number: An Irrational Number is a real number that cannot be written as a simple fraction. The popular approximation of Pi, or 22/7 = 3.14159265358793238462643383279 5… You cannot write down a simple fraction that equals Pi. Phi's sequence, also infinite = 1.61803398874989484820…
- Phi: The golden ratio is a number, approximately 1.618, that possesses many interesting properties. It was studied by ancient mathematicians due to its frequent appearance in geometry. Shapes defined by the golden ratio have long been considered aesthetically pleasing in western cultures, reflecting nature's balance between symmetry and asymmetry. The ratio is still used frequently in art and design. The golden ratio is also known as the golden mean, golden section, golden number or divine proportion. It is usually denoted by the Greek letter ϕ (phi).

Avant-Garden:

- Notice a set of pavers in 3 groups – a long line, a short line, and a circle. The longest line represents the circumference of the circle and the short line represents its diameter. If you divide the length of the long line (22 pavers) / short line (7 pavers) = you reach pi, the irrational number.
- Notice the large rectangle made by the various colored tiles, of which three of its corners are touched by a stainless steel spiral. When a square is cut from this rectangle, the remainder is a rectangle of exactly the same proportion. As squares continue to be removed leaving smaller and smaller rectangles, this proportion remains the same. This proportion is the basis for many things found in nature, from the sunflower floret to the nautilus shell.
- The term labyrinth is often used interchangeably with maze, but a maze is a tour puzzle in the form of a complex branching passage with choices of path and direction; while a single-path (unicursal) labyrinth has a single Eulerian path to the center. A labyrinth has an ambiguous through-route to the center and back and is not designed to be difficult to navigate.
- Notice the path of crushed limestone leads you through a course of hedges (formed by the poda carpus shrub) which curve and angle abruptly until reaching the center. The center is marked by the tallest cypress tree on the grounds, a symbol of resurrection. The labyrinth's design is derived from the Labyrinth at Chartres Cathedral in France.

Glass Enigma:

- It is the only irregular tessellation structure of its kind in North America. The Enigma is made up of over 1,000 triangles, each one a slightly different size. Each triangle is double paned glass. The triangles were fabricated by computer controls on robotic cutting tools and identified with bar-coding to keep track for final assembly or replacement.
- A tessellation is a collection of plane figures (triangles) that fills the plane with no overlaps or gaps. A geodesic dome is a spherical or partial-spherical shell structure or lattice shell based on a network of great circles (geodesics) lying on the surface of a sphere. The geodesics intersect to form triangular elements that have local triangular rigidity and also distribute the stress across the entire structure.
- Dalí was a fan of Buckminster Fuller, the inventor of the Geodesic dome. The artist enlisted a student of Fuller to create his own geodesic dome (situated above his Teatre-Museu in Figueres, Spain). At the Dalí Museum this concept is taken a step further with a dome that pours out of the center of the concrete-box structure. This geodesic glass structure, nicknamed the "Glass Enigma," is a 21st century expression of Buckminster Fuller's original design.

Golden Ratio:

- Nature Morte Vivante, 1956, Underlying design of the painting is the harmonic mathematical grid from the study of aesthetic proportions by Matila Ghyka
- The Ecumenical Council, 1960,
- The Hallucinogenic Toreador, Galacidalacidesoxiribunucleicacid (Homage to Crick and Watson), 1968-70,
- Velazquez Painting the Infanta Margarita with the Lights and Shadows of his Own Glory, 1958,
- Venus de Milo with Drawers (and pompoms), 1936.
- Woodworking for Mortals: http://woodworkingformeremortals.com/2011/03/phi-and-golden-rectangle.html

DNA:

- Nature Morte Vivante, 1956, Railing post in the form of the double-helix structure representing the DNA molecule.
- Galacidalacidesoxiribunucleicacid (Homage to Crick and Watson), 1963, DNA molecule represented with an image on the canvas and the title.

Platonic Solids (Cubes):

- Portrait of My Dead Brother, 1963, Cherries form a cube with their stems.
- Gala Contemplating the Mediterranean Sea Which at Twenty Meters Becomes the Portrait of Abraham Lincoln-Homage to Rothko (Second version), 1976, Dali uses repeated squares of color to pixilate the portrait of Abraham Lincoln.
- Galacidalacidesoxiribunucleicacid (Homage to Crick and Watson), 1963, Group of Arab gunmen in "molecular" cube-like formations.

Fractals:

- A curve or geometric figure, each part of which has the same statistical character as the whole. Fractals are useful in modeling structures in which similar patterns recur at progressively smaller scales, and in describing partly random or chaotic phenomena such as crystal growth, fluid turbulence, and galaxy formation.
- Nature Morte Vivante, 1956, Illustrated in the waves becoming repeated geometric shapes.
- The Disintegration of the Persistence of Memory, 1952-54, Illustrated in the geometric structures receding into the background.

Tiling and Tessellations:

- Gala Contemplating the Mediterranean Sea Which at twenty Meters Becomes the Portrait of Abraham Lincoln-Homage to Rothko (Second Version), 1976, Illustrated in the repeated pattern of square tiles creating the illusion of Abraham Lincoln's portrait.

Anamorphic Art:

- Anamorphoses Skull, 1972, Distorted image of a skull corrects itself when reflected in a cylindrical mirrored surface.
- Anamorphoses Clown, 1972, Distorted image of a clown corrects itself when reflected in a cylindrical mirrored surface.
- http://anamorphicart.wordpress.com/page/2/
- You Tube Video (3D iPad-Cylindrical Mirror Optical Illusion) http://www.youtube.com/watch?v=JqyvDOP_vZM

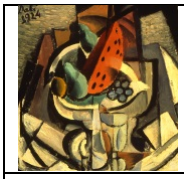
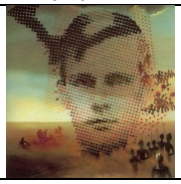
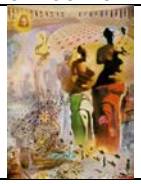
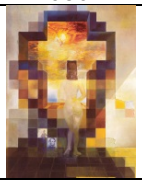
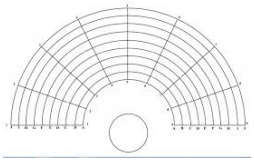
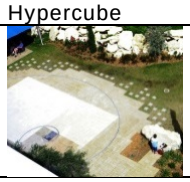
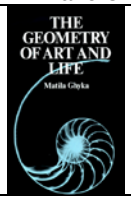
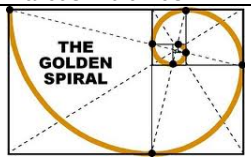
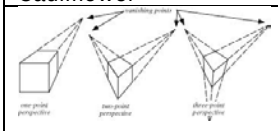
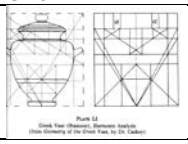
Hypercube:

- Dalí: painting in the fourth dimension.
- http://www.philipcoppens.com/dali.html

General Research Related to Dali and Mathematics:

- Dalí Atomicus, or the Prodigious Adventure of the Lacemaker and the Rhinoceros, Elliot H. King, University of Essex.
- http://sls-2002.caltech.edu/king_paper.html
- Salvador Dalí:
- http://www.abcgallery.com/D/dali/dalibio.html
- Nuclear Mysticism:
- http://www.tufts.edu/programs/mma/fah_188/clifford/Subsections/NuclearMysticism/nuclearmysticism.html
- Nuclear Mysticism Homage to Salvador Dali:
- http://ionamiller.50megs.com/photo5.html

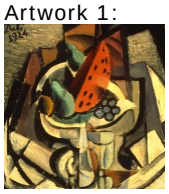
Suggested Illustrations:

				
1. 1924	2. 1926	3. 1952-54	4. 1956	5. 1960
				
6. 1963	7. 1963	8. 1969-70	9. 1976	10. 1958-59
				
11. 1958	12. 1955	13. 1958	14. ca. 1963	15. 1936
				
16. 1972	Polar Grid	Hypercube	The Enigma	Tessellation
				
Helix	Unicursal Labyrinth	Phi Pavers	Pi Pavers (22/7)	Marcus Vitruvius
				
Vitruvian Man	Ghyka	1946	Heisenberg	Golden Rectangle
				
Cauliflower	Nautilus Shell	Sunflower	Alberti	Grid
				
Perspective: 1,2,3 Point	Greek Letter Phi	Greek Letter Pi	Platonic Solid	Stamnos Vase
Inverted Grid	Phi Grid			

Suggested Tour Artworks: (Title, Date, Medium, Scale and Description)

Suggested Number of Artworks per Tour: (Eight to Twelve)

Artwork 1:



Still Life (Sandia), 1924, oil on canvas, 19 ¼ x 19 ¼ in.

- Early geometric organization of space.
- Cubism breaks up an image and simplifies it with geometric shapes and forms.
- Dalí was inspired by Picasso, Braque, Gris and others.

Artwork 2:

Girl with Curls, 1926, oil on panel, 20 x 15 3/4 in.

- Early Vermeer inspired perspective.
- The math of artistic perspective.
- Paintings are simply flat pieces of wood, cloth or paper, etc.
- Artists can create the illusion that you are looking right through a painting.
- Perspective in a landscape has a horizon line, the line between the sky and land.
- There is a point located at a specific place on the horizon line, called the vanishing point.
- Orthogonal lines travel from an object back to the vanishing point and create the illusion of space.
- Dalí takes the traditional rules of perspective and subverts them in this painting.

Artwork 3:

The Disintegration of the Persistence of Memory, 1952-54, oil on canvas, 10 x 13 in.

- Dalí expresses his interest in the exterior world of physics and Werner Karl Heisenberg.
- Dismantled his earlier surrealist masterpiece to reveal a new structure that visualizes quantum mechanics.
- Extreme use of perspective employed in the grid like construction throughout the foreground and extending into the middle ground, also referencing the mathematical concept of fractals.

Artwork 4:

Nature Morte Vivante (Still Life – Fast Moving), 1956, oil on canvas, 49 ¼ x 63 in.

- This is a key painting that shows Dalí's intense interest in the geometry of art, the science of beauty and the spiral form.
- Inspired by Matila Ghyka, a Romanian mathematician, who explored "dynamic symmetry" in art and nature using simple mathematical formulas (such as Phi) to explore natural forms.
- Dalí incorporated several of these symmetrical grids as the compositional basis for many paintings, such as the harmonic rectangle (the Phi rectangle) and the dynamic triangle (from Greek canons of proportion).
- The ideas of geometry of art and life are further expanded with the connection of the Fibonacci spiral; the numerical sequence of the Golden spiral: (1,1,2,3,5,8,13,21…).
- Werner Karl Heisenberg's work on quantum theory is linked with a basic conception of atomic physics.
- DNA double-helix molecular structure represented in the railing post.
- The mathematical concept of fractals is evident in the repeated pattern within the painting of the sea.

Artwork 5:

The Ecumenical Council, 1960, oil on canvas, 118 x 100 in.

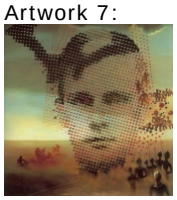
- Matila Ghyka's investigation of proportion leads him to a study in Greek proportion from various Greek vase designs.
- Greek vases have specific mathematic ratios that can be studied in terms of geometry.
- Dalí utilized the analysis of the Greek vase "Stamnos" and used its reversed direction as a compositional basis for this monumental painting.

Artwork 6:

Galacidalacidesoxiribunucleicacid (Homage to Crick and Watson), 1963, oil on canvas, 120 x 163 ½ in.

- Dalí combined his name, the name of his wife Gala, Allah, and Cid Campeador (the feminine Cid) with desoxiribunucleic acid.
- Dalí weaves his beliefs on nuclear mysticism into a complex and often esoteric historical narrative.
- DNA molecule represents the building-block of life (Dr. Francis Crick and Dr. James Watson, 1953).
- Group of Arab gunmen in "molecular" formations in a geometric cube design.

Artwork 7:



Portrait of My Dead Brother, 1963, oil on canvas, 69 x 69 in.

- Dalí's older brother, Salvador, died and Dalí inherited his brother's name.
- Dalí imagined himself as one-half of a double whose unity was irretrievable and kept him in a state of perpetual crisis.
- Cherries joined in a molecular structure of a cube design representing platonic solids.
- Geometric pattern of dots/cherries create his dead brother's imaginary visage.

Artwork 8:

The Hallucinogenic Toreador, 1969-70, oil on canvas, 157 x 118 in.

- Venus figure, the Greek geometric ideal of feminine proportion, is repeated many times and in various ways throughout the painting.
- Geometric grid formed by colored dots and gadflies in multiple locations on the canvas.
- Golden Spiral employed to organize the numerous images in an aesthetic way based on a mathematical ratio (22/7).

Artwork 9:

Gala Contemplating the Mediterranean Sea which at Twenty Meters Becomes the Portrait of Abraham Lincoln-Homage to Rothko (Second Version), 1976, oil and collage on canvas,
99 ¼ x 75 ½ in.

- Carefully calibrated square cells that form a complex network of multiple images and two for one optical illusions.
- Scientific American article, 1973, by Leon D. Harmon titled "The Recognition of Faces," included the transformed and digitized version of Anthony Berger's 1864 portrait of Lincoln.
- Dalí understood the implications of Harmon's research for the growing fields of neuroscience and computer imaging.
- This painting is designed with a grid-like pattern of squares of color employing the mathematical concepts of tessellations, tiling and platonic solids.

Artwork 10:

The Discovery of America by Christopher Columbus, 1958-59, oil on canvas,
161 ½ x 122 1/8 in.

- The structure of the painting is based on the harmonic rectangle calculated by Matila Ghyka in *The Geometry of Art and Life*.
- Two symmetric mirrored images of Dalí's *Christ of Saint John of the Cross*, 1951.
- Repeated linear pattern of crosses, staffs and weapons create movement throughout the canvas.
- One-point perspective employed in the angled crosses and shadows to create the illusion of depth.

Artwork 11:

Leda Atomica design drawing, 1947.

- Renaissance inspired perspective using architectural elements as well as classical figure proportions, based on the Golden Ratio.
- Intentional use of a pentagon whose angles intersect with a circle to create the optimum aesthetic organization of visual elements.
- Reminiscent of Leonardo da Vinci's *Vitruvian Man*, 1490, based on the work of the architect Vitruvius.

Artwork 12:

The Sacrament of the Last Supper, 1955, oil on canvas, 105 x 66 in.

- This painting is designed with connections to the number twelve including: the 12 Apostles, Dodecahedrons and references to numerology.

Artwork 13:

Velazquez Painting the Infanta Margarita with the Lights and Shadows of His Own Glory, 1958, oil on canvas,
60 ½ x 36 ¼ in.

- Dalí viewed Velázquez as a quintessential realist whose “impressionistic” approach to color and form presaged developments in modern art.
- Between 1958 and 1982 Dalí executed a number of paintings after works by Velázquez.
- Dalí locates Velázquez in a continuum that spans the entire history of Spanish painting, from the Golden to the Atomic Age.
- Dalí’s fragmentation of the figure and the overall design of light and shadow speak not only to geometry, but also Nuclear Mysticism and fractals.

Artwork 14:

Fifty Abstract Paintings Which Seen from Two Yards Change into Three Lenins Masquerading as Chinese and as Seen from Six Yards Appear as the Head of a Royal Bengal Tiger, ca. 1963, oil on canvas.

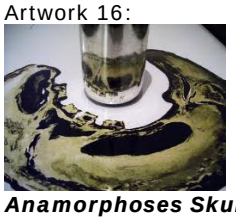
- Each of the fifty panels of this painting is a separate abstract painting, which as seen from two yards away, change into three Lenins masquerading as Chinese.
- When seen from six yards away, the whole painting comes together to appear as the head of a royal tiger.
- This is an excellent use of geometry, based on the square, employing the mathematical concept of tiling.

Artwork 15:

Venus de Milo with Drawers (and pompoms), 1936, plaster cast, 39 ½ x 11 5/8 x 11 in., sculpture.

- As a child, Dalí’s first sculpture was a clay copy of the Venus de Milo.
- Greek marble sculpture of the goddess of love.
- This armless figure has become the icon of classical female beauty based on the golden ratio.
- Dalí cuts six drawers into Venus, transforming the Greek goddess into a piece of living furniture and dividing the golden rectangle with six rectangular cut outs.
- Simple white surface is complemented by elegant fur knobs, a tribute to her beauty and erotic potential.
- The drawers are a metaphor for the way Freudian psychoanalysis opens the hidden areas of the unconscious.

Artwork 16:



Anamorphoses Skull, 1972.

- Distorted image of a skull corrects itself when reflected in a mirrored cylindrical surface.
- Original image is created within a polar grid on a flat surface.
- Mirrored cylinder is placed at the vanishing point of the polar grid perpendicular to the flat surface.
- Dalí has forced us to see the reflected distortion of the actual image as reality, playing off the opposing perceptions of the two hemispheres of our brain.

Vocabulary:

Alberti grid Buckminster Fuller Fibonacci sequence Geodesic dome Geometry Glass enigma Golden rectangle Golden spiral Horizon line Hyper cube Irrational number Irregular tessellation Labyrinth Leon Battista Alberti Marcus Vitruvius Pollio Matila Ghyka Maze Orthogonal lines Perspective Phi Pi Tessellation Tiling Vanishing point Werner Karl Heisenberg

Declarative Knowledge: (Students/Group will Know/Understand)

Students/group will know/understand: important mathematic and scientific concepts relating to Dali's artwork.
Students/group will know/understand: how relevant periods or styles of art change over time because of new mathematic or scientific discoveries.
Students/group will know/understand: how the individuals, time frames and related mathematic and scientific discoveries relate to Dalí's personal life and beliefs.

Procedural Knowledge: (Students/Group will be able to do)

Students/group will be able to: discuss important mathematic and scientific concepts (Fibonacci sequence, Golden ratio, points, lines, angles, planes, polygons, polyhedra, platonic solids, etc.) relating to Dali's artwork.
Students/group will be able to: identify and describe, verbally and in writing, how Dalí's periods or styles of art changed over time because of new mathematic or scientific discoveries.
Students/group will be able to: relate important facts concerning how the individuals, time frames, and related mathematic and scientific discoveries relate to Dalí's personal life and beliefs.

NGSSS: Next Generation Sunshine State Standards (Florida)

Discrete Mathematics (**D**) and Geometry (**G**)

http://tools.fcit.usf.edu/sss/

MA.912.D.11	Sequences and Series – Students define and use arithmetic and geometric sequences and series. Benchmark: 5 (MA.912.D.11.5) Explore and use other sequences found in nature such as the Fibonacci sequence and the golden ratio.
MA.912.G.1	Points, Lines, Angles, and Planes – Students understand geometric concepts, applications, and their representations with coordinate systems. They find lengths and midpoints of line segments, slopes, parallel and perpendicular lines, and equations of lines. Using a compass and straightedge, patty paper, a drawing program or other techniques, students also construct lines and angles, explaining and justifying the processes they use.
MA.912.G.2	Polygons – Students identify and describe polygons (triangles, quadrilaterals, pentagons, hexagons, etc.), using terms such as regular, convex, and concave. They find measures of angles, sides, perimeters, and areas of polygons, justifying their methods. They apply transformations to polygons. They relate geometry to algebra by using coordinate geometry to determine transformations. Students use algebraic reasoning to determine congruence, similarity, and symmetry. Students create and verify tessellations of the plane using polygons.
MA.912.G.7	Polyhedra and Other Solids – Students describe and make regular and non-regular polyhedral (cube, pyramid, tetrahedron, octahedron, etc.). They explore relationships among the faces, edges, and vertices of polyhedra. They describe sets of points on spheres, using terms such as great circle. They describe symmetries of solids and understand the properties of congruent and similar solids.

Formative Assessments:

1. Observation of student engagement.
2. Monitoring student progress and “Teachable Moments.”
3. Discussion participation and responses.

Summative Assessments: (Scoring Scales/Rubrics)

LEARNING GOAL(S)	4 COMPLEX Personal Application	3 TARGET Success for all Students	2 SIMPLER Limited Success	1 PARTIAL Minimal Success	0 NO SUCCESS Unsatisfactory
Able to discuss important mathematic and scientific concepts relating to Dalí's work.	Able to discuss all mathematic and scientific concepts with deep understanding.	Able to discuss most mathematic and scientific concepts.	Able to discuss some mathematic and scientific concepts.	Able to discuss few mathematic and scientific concepts.	No evidence of concepts.
Able to identify and describe, verbally and in writing, how and why Dalí's styles changed over time.	Able to identify and describe how and why Dalí's styles changed.	Able to identify and describe how and why most of Dalí's styles changed.	Able to identify and describe how and why some of Dalí's styles changed.	Able to identify and describe how and why few of Dalí's styles changed.	No evidence of reasoning.
Able to relate important facts about individuals, time frames and discoveries affecting Dalí's personal style and beliefs.	Able to relate in depth about individuals, time frames and discoveries affecting Dalí's style and beliefs.	Able to relate most important facts about individuals, time frames and discoveries affecting Dalí.	Able to relate some facts about individuals, time frames and discoveries affecting Dalí.	Able to relate few facts about individuals, time frames or discoveries affecting Dalí.	No evidence of understanding.

REFERENCE SCALE/RUBRIC USED TO ASSESS: Visual Art, Design or any Creative Endeavor.

FINE ART SCALE (RUBRIC)	4 COMPLEX Personal Application	3 TARGET Success for all Students	2 SIMPLER Limited Success	1 PARTIAL Minimal Success	0 NO SUCCESS Unsatisfactory
KNOWLEDGE	Uses basic directions and concepts of the assignment in a unique way.	All basic directions and concepts of the assignment clearly evident.	Uses most assignment specific directions and concepts.	Minimal assignment specific directions and concepts evident.	No evidence of knowledge.
REASONING	Connecting information in introspective, logical and sequential choices throughout entire creative process.	Connecting information in logical and sequential choices throughout entire creative process.	Connecting some information in choices throughout entire creative process.	Minimal connection of information in choices throughout entire creative process.	No evidence of reasoning.
TECHNICAL SKILLS	Demonstrates high level of expertise in techniques appropriately employed.	Uses all relevant techniques appropriately.	Uses most relevant techniques appropriately.	Minimal use of appropriate and relevant techniques.	No evidence of technical skills.
CREATIVITY	Exceptional evidence of personal style continued throughout creative process and product.	Solid evidence of personal style continued throughout creative process and product.	Some evidence of personal style continued throughout creative process and product.	Limited evidence of personal style continued throughout creative process and product.	No evidence of creativity.

ADDITIONAL REFERENCE MATERIAL:

Elements of Art:

Line, Shape, Color, Value, Form, Texture, Space.

Principles of Design:

Balance, Contrast, Emphasis, Movement, Pattern, Rhythm, Unity.

National Core Art Standards:

www.nationalartstandards.org

Creating, Performing/Presenting/Producing, Responding, Connecting.

Anchor Standards:

Creating:

1. Generate and conceptualize artistic ideas and work.
2. Organize and develop artistic ideas and work.
3. Refine and complete artistic work.

Performing/Presenting/Producing:

4. Analyze, interpret, and select artistic work for presentation.
5. Develop and refine artistic work for presentation.
6. Convey meaning through the presentation of artistic work.

Responding:

7. Perceive and analyze artistic work.
8. Interpret intent and meaning in artistic work.
9. Apply criteria to evaluate artistic work.

Connecting:

10. Synthesize and relate knowledge and personal experiences to make art.
11. Relate artistic ideas and works with societal, cultural and historical context to deepen understanding.

Critical Thinking:

Knowledge, Comprehension, Application, Analysis, Synthesis, Evaluation.

Bloom's Taxonomy:

Remembering, Understanding, Applying, Analyzing, Evaluating, Creating.

Marzano's Taxonomy:

Retrieval	Recognizing, recalling, executing.
Comprehension	Integrating, symbolizing.
Analysis	Matching, classifying, analyzing errors, generalizing, specifying.
Knowledge Utilization	Decision making, problem solving, experimenting, investigating.

Feldman's Model of Art Criticism (1981):

Description	What do you see in this work?
Analysis	How is the work organized?
Interpretation	What is the work about?
Judgment	Is the work successful? Why?

Anderson's Model of Art Criticism (1988):

Reaction	What is it?
Description	What does the work show? How, why, where was it made?
Interpretation	What is the work about? How do we know?
Evaluation	Is the work well done? How do we decide?

NGSSS: (Standard Numbers/Standards/Taxonomy Levels)

Next Generation Sunshine State Standards (Florida)

http://tools.fcit.usf.edu/sss/

Mathematics: Algebra (A.1-10), Calculus (C.1-5), Discrete Mathematics (D.1-11), Financial Literacy (F.1-5), Geometry (G.1-8), Probability (P.1-3), Statistics (S.1-5), Trigonometry (T.1-5)

Mathematics 9-12

Algebra

Standard 1: Real and Complex Number Systems - Students expand and deepen their understanding of real and complex numbers by comparing expressions and performing arithmetic computations, especially those involving square roots and exponents. They use the properties of real numbers to simplify algebraic expressions and equations, and they convert between different measurment units using dimensional analysis. (MA.912.A.1)

Benchmark: 1. * Know equivalent forms of real numbers (including integer exponents and radicals, per cents, scientific notation, absolute value, rational numbers, irrational numbers). (MA.912.A.1.1)
Benchmark: 2. * Compare real number expressions. (MA.912.A.1.2)
Benchmark: 3. * Simplify real number expressions using the laws of exponents. (MA.912.A.1.3)
Benchmark: 4. * Perform operations on real numbers (including integer exponents, radicals, percents, scientific notation, absolute value, rational numbers, and irrational numbers) using multi-step and real-world problems. (MA.912.A.1.4)
Benchmark: 5. * Use dimensional (unit) analysis to perform conversions between units of measure, including rates. (MA.912.A.1.5)
Benchmark: 6. Identify the real and imaginary parts of complex numbers and perform basic operations. (MA.912.A.1.6)
Benchmark: 7. Represent complex numbers geometrically. (MA.912.A.1.7)
Benchmark: 8. Use the zero product property of real numbers in a variety of contexts to identify solutions to equations. (MA.912.A.1.8)

Standard 2: Relations and Functions - Students draw and interpret graphs of relations. They understand the notation and concept of a function, find domains and ranges, and link equations to functions. (MA.912.A.2)

Benchmark: 1. * Create a graph to represent a real-world situation. (MA.912.A.2.1)
Benchmark: 2. * Interpret a graph representing a real-world situation. (MA.912.A.2.2)
Benchmark: 3. * Describe the concept of a function, use function notation, determine whether a given relation is a function, and link equations to functions. (MA.912.A.2.3)
Benchmark: 4. * Determine the domain and range of a relation. (MA.912.A.2.4)
Benchmark: 5. Graph absolute value equations and inequalities in two variables. (MA.912.A.2.5)
Benchmark: 6. Identify and graph common functions (including but not limited to linear, rational, quadratic, cubic, radical, absolute value). (MA.912.A.2.6)
Benchmark: 7. Perform operations (addition, subtraction, division and multiplication) of functions algebraically, numerically, and graphically. (MA.912.A.2.7)
Benchmark: 8. Determine the composition of functions. (MA.912.A.2.8)
Benchmark: 9. Recognize, interpret, and graph functions defined piece-wise, with and without technology. (MA.912.A.2.9)
Benchmark: 10. Describe and graph transformations of functions (MA.912.A.2.10)
Benchmark: 11. Solve problems involving functions and their inverses. (MA.912.A.2.11)
Benchmark: 12. * Solve problems using direct, inverse, and joint variations. (MA.912.A.2.12)
Benchmark: 13. Solve real-world problems involving relations and functions. (MA.912.A.2.13)

Standard 3: Linear Equations and Inequalities - Students solve linear equations and inequalities. (MA.912.A.3)

Benchmark: 1. * Solve linear equations in one variable that include simplifying algebraic expressions. (MA.912.A.3.1)
Benchmark: 2. * Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality. (MA.912.A.3.2)
Benchmark: 3. * Solve literal equations for a specified variable. (MA.912.A.3.3)
Benchmark: 4. * Solve and graph simple and compound inequalities in one variable and be able to justify each step in a solution. (MA.912.A.3.4)
Benchmark: 5. * Symbolically represent and solve multi-step and real-world applications that involve linear equations and inequalities. (MA.912.A.3.5)
Benchmark: 6. Solve and graph the solutions of absolute value equations and inequalities with one variable. (MA.912.A.3.6)
Benchmark: 7. * Rewrite equations of a line into slope-intercept form and standard form. (MA.912.A.3.7)
Benchmark: 8. * Graph a line given any of the following information: a table of values, the x- and y-intercepts, two points, the slope and a point, the equation of the line in slope-intercept form, standard form, or point-slope form. (MA.912.A.3.8)
Benchmark: 9. * Determine the slope, x-intercept, and y-intercept of a line given its graph, its equation, or two points on the line. (MA.912.A.3.9)
Benchmark: 10. * Write an equation of a line given any of the following information: two points on the line, its slope and one point

on the line, or its graph. Also, find an equation of a new line parallel to a given line, or perpendicular to a given line, through a given point on the new line. (MA.912.A.3.10)
Benchmark: 11. * Write an equation of a line that models a data set and use the equation or the graph to make predictions. Describe the slope of the line in terms of the data, recognizing that the slope is the rate of change. (MA.912.A.3.11)
Benchmark: 12. * Graph a linear equation or inequality in two variables with and without graphing technology. Write an equation or inequality represented by a given graph. (MA.912.A.3.12)
Benchmark: 13. * Use a graph to approximate the solution of a system of linear equations or inequalities in two variables with and without technology. (MA.912.A.3.13)
Benchmark: 14. * Solve systems of linear equations and inequalities in two and three variables using graphical, substitution, and elimination methods. (MA.912.A.3.14)
Benchmark: 15. * Solve real-world problems involving systems of linear equations and inequalities in two and three variables. (MA.912.A.3.15)

Standard 4: Polynomials - Students perform operations on polynomials. They find factors of polynomials, learning special techniques for factoring quadratics. They understand the relationships among the solutions of polynomial equations, the zeros of a polynomial function, the x-intercepts of a graph, and the factors of a polynomial. (MA.912.A.4)

Benchmark: 1. * Simplify monomials and monomial expressions using the laws of integral exponents. (MA.912.A.4.1)
Benchmark: 2. * Add, subtract, and multiply polynomials. (MA.912.A.4.2)
Benchmark: 3. * Factor polynomial expressions. (MA.912.A.4.3)
Benchmark: 4. * Divide polynomials by monomials and polynomials with various techniques, including synthetic division. (MA.912.A.4.4)
Benchmark: 5. Graph polynomial functions with and without technology and describe end behavior. (MA.912.A.4.5)
Benchmark: 6. Use theorems of polynomial behavior (including but not limited to the Fundamental Theorem of Algebra, Remainder Theorem, the Rational Root Theorem, Descartes (MA.912.A.4.6)
Benchmark: 7. Write a polynomial equation for a given set of real and/or complex roots. (MA.912.A.4.7)
Benchmark: 8. Describe the relationships among the solutions of an equation, the zeros of a function, the x-intercepts of a graph, and the factors of a polynomial expression, with and without technology. (MA.912.A.4.8)
Benchmark: 9. Use graphing technology to find approximate solutions for polynomial equations. (MA.912.A.4.9)
Benchmark: 10. Use polynomial equations to solve real-world problems. (MA.912.A.4.10)
Benchmark: 11. Solve a polynomial inequality by examining the graph with and without the use of technology. (MA.912.A.4.11)
Benchmark: 12. Apply the Binomial Theorem. (MA.912.A.4.12)

Standard 5: Rational Expressions and Equations - Students simplify rational expressions and solve rational equations using what they have learned about factoring polynomials. (MA.912.A.5)

Benchmark: 1. * Simplify algebraic ratios. (MA.912.A.5.1)
Benchmark: 2. Add, subtract, multiply, and divide rational expressions. (MA.912.A.5.2)
Benchmark: 3. Simplify complex fractions. (MA.912.A.5.3)
Benchmark: 4. * Solve algebraic proportions. (MA.912.A.5.4)
Benchmark: 5. Solve rational equations. (MA.912.A.5.5)
Benchmark: 6. Identify removable and non-removable discontinuities and vertical, horizontal, and oblique asymptotes of a graph of a rational function, find the zeros, and graph the function. (MA.912.A.5.6)Benchmark: 7. Solve real-world problems involving rational equations (mixture, distance, work, interest, and ratio). (MA.912.A.5.7)

Standard 6: Radical Expressions and Equations - Students simplify and perform operations on radical expressions and equations. They also rationalize square root expressions and understand and use the concepts of negative and rational exponents. They add, subtract, multiply, divide, and simplify radical expressions and expressions with rational exponents. Students will solve radical equations and equations with terms that have rational exponents. (MA.912.A.6)

Benchmark: 1. * Simplify radical expressions. (MA.912.A.6.1)
Benchmark: 2. * Add, subtract, multiply and divide radical expressions (square roots and higher). (MA.912.A.6.2)
Benchmark: 3. Simplify expressions using properties of rational exponents. (MA.912.A.6.3)
Benchmark: 4. Convert between rational exponent and radical forms of expressions. (MA.912.A.6.4)
Benchmark: 5. Solve equations that contain radical expressions. (MA.912.A.6.5)

Standard 7: Quadratic Equations - Students draw graphs of quadratic functions. They solve quadratic equations and solve these equations by factoring, completing the square and by using the quadratic formula. They also use graphing calculators to find approximate solutions of quadratic equations. (MA.912.A.7)

Benchmark: 1. * Graph quadratic equations with and without graphing technology. (MA.912.A.7.1)
Benchmark: 2. * Solve quadratic equations over the real numbers by factoring, and by using the quadratic formula. (MA.912.A.7.2)
Benchmark: 3. Solve quadratic equations over the real numbers by completing the square. (MA.912.A.7.3)
Benchmark: 4. Use the discriminant to determine the nature of the roots of a quadratic equation. (MA.912.A.7.4)
Benchmark: 5. Solve quadratic equations over the complex number system. (MA.912.A.7.5)
Benchmark: 6. Identify the axis of symmetry, vertex, domain, range and intercept(s) for a given parabola. (MA.912.A.7.6)
Benchmark: 7. Solve non-linear systems of equations with and without using technology. (MA.912.A.7.7)

Benchmark: 8. * Use quadratic equations to solve real-world problems. (MA.912.A.7.8)
Benchmark: 9. Solve optimization problems. (MA.912.A.7.9)
Benchmark: 10. Use graphing technology to find approximate solutions of quadratic equations. (MA.912.A.7.10)

Standard 8: Logarithmic and Exponential Functions - Students understand the concepts of logarithmic and exponential functions. They graph exponential functions and solve problems of growth and decay. They understand the inverse relationship between exponents and logarithms and use it to prove laws of logarithms and to solve equations. They convert logarithms between bases and simplify logarithmic expressions. (MA.912.A.8)

Benchmark: 1. Define exponential and logarithmic functions and determine their relationship. (MA.912.A.8.1)
Benchmark: 2. Define and use the properties of logarithms to simplify logarithmic expressions and to find their approximate values. (MA.912.A.8.2)
Benchmark: 3. Graph exponential and logarithmic functions. (MA.912.A.8.3)
Benchmark: 4. Prove laws of logarithms. (MA.912.A.8.4)
Benchmark: 5. Solve logarithmic and exponential equations. (MA.912.A.8.5)
Benchmark: 6. Use the change of base formula. (MA.912.A.8.6)
Benchmark: 7. Solve applications of exponential growth and decay. (MA.912.A.8.7)

Standard 9: Conic Sections - Students write equations and draw graphs of conic sections (circle, ellipse, parabola, and hyperbola), thus relating an algebraic representation to a geometric one. (MA.912.A.9)

Benchmark: 1. Write the equations of conic sections in standard form and general form, in order to identify the conic section and to find its geometric properties (foci, asymptotes, eccentricity, etc.). (MA.912.A.9.1)
Benchmark: 2. Graph conic sections with and without using graphing technology. (MA.912.A.9.2)
Benchmark: 3. Solve real-world problems involving conic sections. (MA.912.A.9.3)

Standard 10: Mathematical Reasoning and Problem Solving - In a general sense, all of mathematics is problem solving. In all of their mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. (MA.912.A.10)

Benchmark: 1. * Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guess- and-check, solving a simpler problem, writing an equation, working backwards, and create a table. (MA.912.A.10.1)
Benchmark: 2. * Decide whether a solution is reasonable in the context of the original situation. (MA.912.A.10.2)
Benchmark: 3. * Decide whether a given statement is always, sometimes, or never true (statements involving linear or quadratic expressions, equations, or inequalities rational or radical expressions or logarithmic or exponential functions). (MA.912.A.10.3)
Benchmark: 4. Use counterexamples to show that statements are false. (MA.912.A.10.4)

Calculus

Standard 1: Limits and Continuity - Students develop an understanding of the concept of limit by estimating limits graphically and numerically, and evaluating limits analytically. They extend the idea of a limit to one-sided limits and limits at infinity. They use limits to define and understand the concept of continuity, decide whether a function is continuous at a point, and find types of discontinuities. They understand and apply continuity theorems. (MA.912.C.1)

Benchmark: 1. Understand the concept of limit and estimate limits from graphs and tables of values. (MA.912.C.1.1)
Benchmark: 2. Find limits by substitution. (MA.912.C.1.2)
Benchmark: 3. Find limits of sums, differences, products, and quotients. (MA.912.C.1.3)
Benchmark: 4. Find limits of rational functions that are undefined at a point. (MA.912.C.1.4)
Benchmark: 5. Find one-sided limits. (MA.912.C.1.5)
Benchmark: 6. Find limits at infinity. (MA.912.C.1.6)
Benchmark: 7. Decide when a limit is infinite and use limits involving infinity to describe asymptotic behavior. (MA.912.C.1.7)

Benchmark: 8. Find special limits such as $\lim_{x \to 0} \frac{\sin x}{x}$. (MA.912.C.1.8)
Benchmark: 9. Understand continuity in terms of limits. (MA.912.C.1.9)
Benchmark: 10. Decide if a function is continuous at a point. (MA.912.C.1.10)
Benchmark: 11. Find the types of discontinuities of a function. (MA.912.C.1.11)
Benchmark: 12. Understand and use the Intermediate Value Theorem on a function over a closed interval. (MA.912.C.1.12)
Benchmark: 13. Understand and apply the Extreme Value Theorem: If f(x) is continuous over a closed interval, then f has a maximum and a minimum on the interval. (MA.912.C.1.13)

Standard 2: Differential Calculus - Students develop an understanding of the derivative as an instantaneous rate of change, using geometrical, numerical, and analytical methods. They use this definition to find derivatives of algebraic and transcendental functions and combinations of these functions (using, for example, sums, composites, and inverses). Students find second and higher order

derivatives. They understand and use the relationship between differentiability and continuity. They understand and apply the Mean Value Theorem. Students find derivatives of algebraic, trigonometric, logarithmic, and exponential functions. They find derivatives of sums, products, and quotients, and composite and inverse functions. They find derivatives of higher order and use logarithmic differentiation and the Mean Value Theorem. (MA.912.C.2)

Benchmark: 1. Understand the concept of derivative geometrically, numerically, and analytically, and interpret the derivative as an instantaneous rate of change, or as the slope of the tangent line. (MA.912.C.2.1)
Benchmark: 2. State, understand, and apply the definition of derivative. (MA.912.C.2.2)
Benchmark: 3. Find the derivatives of functions, including algebraic, trigonometric, logarithmic, and exponential functions. (MA.912.C.2.3)
Benchmark: 4. Find the derivatives of sums, products, and quotients. (MA.912.C.2.4)
Benchmark: 5. Find the derivatives of composite functions, using the Chain Rule. (MA.912.C.2.5)
Benchmark: 6. Find the derivatives of implicitly-defined functions. (MA.912.C.2.6)
Benchmark: 7. Find derivatives of inverse functions. (MA.912.C.2.7)
Benchmark: 8. Find second derivatives and derivatives of higher order. (MA.912.C.2.8)
Benchmark: 9. Find derivatives using logarithmic differentiation. (MA.912.C.2.9)
Benchmark: 10. Understand and use the relationship between differentiability and continuity. (MA.912.C.2.10)
Benchmark: 11. Understand and apply the Mean Value Theorem. (MA.912.C.2.11)

Standard 3: Applications of Derivatives - Students apply what they learn about derivatives to find slopes of curves and the related tangent lines. They analyze and graph functions, finding where they are increasing or decreasing, their maximum and minimum points, their points of inflection, and their concavity. They solve optimization problems, find average and instantaneous rates of change (including velocities and accelerations), and model rates of change. Students find slopes and equations of tangent lines, maximum and minimum points, and points of inflection. They solve optimization problems and find rates of change. (MA.912.C.3)

Benchmark: 1. Find the slope of a curve at a point, including points at which there are vertical tangent lines and no tangent lines. (MA.912.C.3.1)
Benchmark: 2. Find an equation for the tangent line to a curve at a point and a local linear approximation. (MA.912.C.3.2)
Benchmark: 3. Decide where functions are decreasing and increasing. Understand the relationship between the increasing and decreasing behavior of *f* and the sign of *f*″. (MA.912.C.3.3)
Benchmark: 4. Find local and absolute maximum and minimum points. (MA.912.C.3.4)
Benchmark: 5. Find points of inflection of functions. Understand the relationship between the concavity of *f* and the sign of *f*″. Understand points of inflection as places where concavity changes. (MA.912.C.3.5)
Benchmark: 6. Use first and second derivatives to help sketch graphs. Compare the corresponding characteristics of the graphs of *f*, *f*′, and *f*″. (MA.912.C.3.6)
Benchmark: 7. Use implicit differentiation to find the derivative of an inverse function. (MA.912.C.3.7)
Benchmark: 8. Solve optimization problems. (MA.912.C.3.8)
Benchmark: 9. Find average and instantaneous rates of change. Understand the instantaneous rate of change as the limit of the average rate of change. Interpret a derivative as a rate of change in applications, including velocity, speed, and acceleration. (MA.912.C.3.9)
Benchmark: 10. Find the velocity and acceleration of a particle moving in a straight line. (MA.912.C.3.10)
Benchmark: 11. Model rates of change, including related rates problems. (MA.912.C.3.11)
Benchmark: 12. Solve problems using the Newton-Raphson method. (MA.912.C.3.12)

Standard 4: Integral Calculus - Students understand that integration is used to find areas and they evaluate integrals using rectangular approximations. From this, they develop the idea that integration is the inverse operation to differentiation (MA.912.C.4)

Benchmark: 1. Use rectangle approximations to find approximate values of integrals. (MA.912.C.4.1)
Benchmark: 2. Calculate the values of Riemann Sums over equal subdivisions using left, right, and midpoint evaluation points. (MA.912.C.4.2)
Benchmark: 3. Interpret a definite integral as a limit of Riemann sums. (MA.912.C.4.3)
Benchmark: 4. Interpret a definite integral of the rate of change of a quantity over an interval as the change of the quantity over the interval. That is, $\int_a^b f'(x)dx = f(b)$ (MA.912.C.4.4)
Benchmark: 5. Use the Fundamental Theorem of Calculus to evaluate definite and indefinite integrals and to represent particular antiderivatives. Perform analytical and graphical analysis of functions so defined. (MA.912.C.4.5)
Benchmark: 6. Use these properties of definite integrals:

- $\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx$
- $\int_a^b k$ (MA.912.C.4.6)

Benchmark: 7. Use integration by substitution (or change of variable) to find values of integrals. (MA.912.C.4.7)
Benchmark: 8. Use Riemann Sums, the Trapezoidal Rule, and technology to approximate definite integrals of functions represented algebraically, geometrically, and by tables of values. (MA.912.C.4.8)

Standard 5: Applications of Integration - Students apply what they learn about integrals to finding velocities from accelerations, solving separable differential equations, and finding areas and volumes. They also apply integration to model and solve problems in physics, biology, economics, etc. Students find velocity functions and position functions from their derivatives, solve separable differential equations, and use definite integrals to find areas and volumes. (MA.912.C.5)

Benchmark: 1. Find specific antiderivatives using initial conditions, including finding velocity functions from acceleration functions, finding position functions from velocity functions, and solving applications related to motion along a line. (MA.912.C.5.1)
Benchmark: 2. Solve separable differential equations and use them in modeling. (MA.912.C.5.2)
Benchmark: 3. Solve differential equations of the form $\frac{dy}{dt}$ = ky as applied to growth and decay problems. (MA.912.C.5.3)
Benchmark: 4. Use slope fields to display a graphic representation of the solution to a differential equation and locate particular solutions to the equation. (MA.912.C.5.4)
Benchmark: 5. Use definite integrals to find the area between a curve and the x-axis, or between two curves. (MA.912.C.5.5)
Benchmark: 6. Use definite integrals to find the average value of a function over a closed interval. (MA.912.C.5.6)
Benchmark: 7. Use definite integrals to find the volume of a solid with known cross-sectional area, including solids of revolution. (MA.912.C.5.7)
Benchmark: 8. Apply integration to model and solve problems in physical, biological, and social sciences. (MA.912.C.5.8)

Discrete Mathematics

Standard 1: Recursion - Students understand and apply recursive methods to solve problems, including the use of finite differences. (MA.912.D.1)

Benchmark: 1. Use recursive and iterative thinking to solve problems, including identification of patterns, population growth and decline, and compound interest. (MA.912.D.1.1)
Benchmark: 2. Use finite differences to solve problems and to find explicit formulas for recurrence relations. (MA.912.D.1.2)
Benchmark: 3. Use mathematical induction to prove various concepts in number theory (such as sums of infinite integer series, divisibility statements, and parity statements), recurrence relations, and other applications. (MA.912.D.1.3)

Standard 2: Graph Theory - Students understand how graphs of vertices joined by edges can model relationships and be used to solve various problems with relation to directed graphs, weighted graphs, networks, tournaments, transportation flows, matching, and coverage. (MA.912.D.2)

Benchmark: 1. Use Euler and Hamilton cycles and paths in graphs to solve routing problems. (MA.912.D.2.1)
Benchmark: 2. Use critical path analysis to solve scheduling problems. (MA.912.D.2.2)
Benchmark: 3. Use graph coloring techniques to solve problems. (MA.912.D.2.3)
Benchmark: 4. Use spanning trees, rooted trees, binary trees, and decision trees to solve problems. (MA.912.D.2.4)
Benchmark: 5. Use bin-packing techniques to solve problems concerning optimizing resource usage. (MA.912.D.2.5)

Standard 3: Social Choice - Students analyze election data to evaluate different election methods and use weighted voting techniques to decide voting power within a group. They understand and use fair division techniques to solve apportionment problems. (MA.912.D.3)

Benchmark: 1. Use election theory techniques to analyze election data. (MA.912.D.3.1)
Benchmark: 2. Use weighted voting techniques to decide voting power within a group. (MA.912.D.3.2)
Benchmark: 3. Use fair division techniques to divide continuous objects. (MA.912.D.3.3)
Benchmark: 4. Use fair division techniques to solve apportionment problems. (MA.912.D.3.4)

Standard 4: Linear Programming - Students understand how to use linear programming and coordinate geometry to solve simple linear optimization problems. (MA.912.D.4)

Benchmark: 1. Solve maximal profit/minimal cost problems. (MA.912.D.4.1)

Standard 5: Game Theory - Students understand and use game theory methods to solve strictly determined games and non-strictly determined games. (MA.912.D.5)

Benchmark: 1. Use game theory to solve strictly determined games. (MA.912.D.5.1)
Benchmark: 2. Use game theory to solve non-strictly determined games. (MA.912.D.5.2)

Standard 6: Logic - Students develop an understanding of the fundamentals of propositional logic, arguments, and methods of proof. (MA.912.D.6)

Benchmark: 1. Use truth tables to determine truth values of propositional statements. (MA.912.D.6.1)
Benchmark: 2. * Find the converse, inverse, and contrapositive of a statement. (MA.912.D.6.2)
Benchmark: 3. Determine whether two propositions are logically equivalent. (MA.912.D.6.3)
Benchmark: 4. * Use methods of direct and indirect proof and determine whether a short proof is logically valid. (MA.912.D.6.4)
Benchmark: 5. * Identify and give examples of:

- undefined terms;
- axioms;
- theorems;
- inductive and deductive proofs; and,
- inductive and deductive reasoning. (MA.912.D.6.5)Benchmark: 6. Construct logical arguments using laws of detachment (modus ponens), syllogism, tautology, and contradiction; judge the validity of arguments, and give counterexamples to disprove statements. (MA.912.D.6.6)

Benchmark: 7. Use applications of the universal and existential quantifiers to propositional statements. (MA.912.D.6.7)

Standard 7: Set Theory - Students operate with sets and use set theory to solve problems. (MA.912.D.7)

Benchmark: 1. * Perform set operations such as union and intersection, complement, and cross product. (MA.912.D.7.1)
Benchmark: 2. * Use Venn diagrams to explore relationships and patterns, and to make arguments about relationships between sets. (MA.912.D.7.2)

Standard 8: Matrices - Students understand how matrices can be used to store and organize data and to solve systems of equations. They also use matrices to solve Markov chain problems that link present events to future events using probabilities. (MA.912.D.8)

Benchmark: 1. Use matrices to organize and store data. Perform matrix operations (addition, subtraction, scalar multiplication, multiplication). (MA.912.D.8.1)
Benchmark: 2. Use matrix operations to solve problems. (MA.912.D.8.2)
Benchmark: 3. Use row-reduction techniques to solve problems. (MA.912.D.8.3)
Benchmark: 4. Find the inverse of a matrix and use the inverse to solve problems with and without the use of technology. (MA.912.D.8.4)
Benchmark: 5. Use determinants of 2 x 2 and 3 x 3 matrices as well as higher order matrices with and without the use of technology. (MA.912.D.8.5)
Benchmark: 6. Use matrices to solve Markov chain problems that link present events to future events using probabilities. (MA.912.D.8.6)

Standard 9: Vectors - Students recognize vectors in both two- and three-dimensions and that they are represented geometrically and algebraically. Students perform basic operations on vectors, including addition, scalar multiplication, dot product, and cross product. Students solve problems using vectors. (MA.912.D.9)

Benchmark: 1. Demonstrate an understanding of the geometric interpretation of vectors and vector operations including addition, scalar multiplication, dot product and cross product in the plane and in three-dimensional space. (MA.912.D.9.1)
Benchmark: 2. Demonstrate an understanding of the algebraic interpretation of vectors and vector operations including addition, scalar multiplication, dot product and cross product in the plane and in three-dimensional space. (MA.912.D.9.2)
Benchmark: 3. Use vectors to model and solve application problems. (MA.912.D.9.3)

Standard 10: Parametric Equations - Students use parametric equations in two dimensions to model time dependant situations and convert parametric equations to rectangular coordinates and vice-versa. (MA.912.D.10)

Benchmark: 1. Sketch the graph of a curve in the plane represented parametrically, indicating the direction of motion. (MA.912.D.10.1)
Benchmark: 2. Convert from a parametric representation of a plane curve to a rectangular equation, and vice-versa. (MA.912.D.10.2)
Benchmark: 3. Use parametric equations to model applications of motion in the plane. (MA.912.D.10.3)

Standard 11: Sequences and Series - Students define and use arithmetic and geometric sequences and series. (MA.912.D.11)

Benchmark: 1. Define arithmetic and geometric sequences and series. (MA.912.D.11.1)
Benchmark: 2. Use sigma notation to describe series. (MA.912.D.11.2)
Benchmark: 3. Find specified terms of arithmetic and geometric sequences. (MA.912.D.11.3)
Benchmark: 4. Find partial sums of arithmetic and geometric series, and find sums of infinite convergent geometric series. Use Sigma notation where applicable. (MA.912.D.11.4)
Benchmark: 5. Explore and use other sequences found in nature such as the Fibonacci sequence and the golden ratio. (MA.912.D.11.5)

Financial Literacy

Standard 1: Simple and Compound Interest - Simple and Compound Interest (MA.912.F.1)

Benchmark: 1. Explain the difference between simple and compound interest. (MA.912.F.1.1)
Benchmark: 2. Solve problems involving compound interest. (MA.912.F.1.2)
Benchmark: 3. Demonstrate the relationship between simple interest and linear growth. (MA.912.F.1.3)
Benchmark: 4. Demonstrate the relationship between compound interest and exponential growth. (MA.912.F.1.4)

Standard 2: Net Present and Net Future Value (NPV and NFV) - Net Present and Net Future Value (NPV and NFV) (MA.912.F.2)

Benchmark: 1. Calculate the future value of a given amount of money, with and without technology. (MA.912.F.2.1)
Benchmark: 2. Calculate the present value of a certain amount of money for a given length of time in the future, with and without technology. (MA.912.F.2.2)
Benchmark: 3. Use a consumer price index to express dollars in constant terms, with and without technology. (MA.912.F.2.3)
Benchmark: 4. Calculate the present value of an income stream, with and without technology. (MA.912.F.2.4)

Standard 3: Loans and Financing - Students are familiar with and can describe the advantages and disadvantages of short-term purchases, long-term purchases, and mortgages. (MA.912.F.3)

Benchmark: 1. Compare the advantages and disadvantages of using cash versus a credit card. (MA.912.F.3.1)
Benchmark: 2. Analyze credit scores and reports. (MA.912.F.3.2)
Benchmark: 3. Calculate the finance charges and total amount due on a credit card bill. (MA.912.F.3.3)
Benchmark: 4. Compare the advantages and disadvantages of deferred payments. (MA.912.F.3.4)
Benchmark: 5. Calculate deferred payments. (MA.912.F.3.5)
Benchmark: 6. Calculate total cost of purchasing consumer durables over time given different down payments, financing options, and fees. (MA.912.F.3.6) Calculate the following fees associated with a mortgage:

- discount points
- origination fee
- maximum brokerage fee on a net or gross loan
- documentary stamps
- prorated expenses (interest, county and/or city property taxes, and mortgage on an assumed mortgage) (MA.912.F.3.7)

Benchmark: 8. Substitute to solve a variety of mortgage formulas, including but not limited to Front End Ratio, Total Debt-to-Income Ratio, Loan-to-Value Ratio (LTV), Combined Loan-to-Value Ratio (CLTV), and Amount of Interest Paid Over the Life of a Loan. (MA.912.F.3.8)
Benchmark: 9. Calculate the total amount to be paid over the life of a fixed rate loan. (MA.912.F.3.9)
Benchmark: 10. Calculate the effects on the monthly payment in the change of interest rate based on an adjustable rate mortgage. (MA.912.F.3.10)
Benchmark: 11. Calculate the final pay out amount for a balloon mortgage. (MA.912.F.3.11)
Benchmark: 12. Compare the cost of paying a higher interest rate and lower points versus a lower interest rate and more points. (MA.912.F.3.12)
Benchmark: 13. Calculate the total amount paid for the life of a loan for a house including the down payment, points, fees, and interest. (MA.912.F.3.13)
Benchmark: 14. Compare the total cost for a set purchase price using a fixed rate, adjustable rate, and a balloon mortgage. (MA.912.F.3.14)
Benchmark: 15. Interpret the legal description using the metes and bounds; lot and block (plat); government survey; and monument methods. (MA.912.F.3.15)
Benchmark: 16. Estimate real property value using the sales comparison approach, cost-depreciation approach, or the income capitalization approach. (MA.912.F.3.16)
Benchmark: 17. Compare interest rate calculations and annual percentage rate calculations to distinguish between the two rates. (MA.912.F.3.17)

Standard 4: Individual Financial and Investment Planning - Individual Financial and Investment Planning (MA.912.F.4)

Benchmark: 1. Develop personal budgets that fit within various income brackets. (MA.912.F.4.1)
Benchmark: 2. Explain cash management strategies including debit accounts, checking accounts, and savings accounts. (MA.912.F.4.2)
Benchmark: 3. Calculate net worth. (MA.912.F.4.3)
Benchmark: 4. Establish a plan to pay off debt. (MA.912.F.4.4)
Benchmark: 5. Develop and apply a variety of strategies to use tax tables, determine, calculate, and complete yearly federal income tax. (MA.912.F.4.5)

Benchmark: 6. Compare different insurance options and fees. (MA.912.F.4.6)
Benchmark: 7. Compare and contrast the role of insurance as a device to mitigate risk and calculate expenses of various options. (MA.912.F.4.7)
Benchmark: 8. Collect, organize, and interpret data to determine an effective retirement savings plan to meet personal financial goals. (MA.912.F.4.8)
Benchmark: 9. Calculate, compare, and contrast different types of retirement plans, including IRAs, ROTH accounts, and annuities. (MA.912.F.4.9)
Benchmark: 10. Analyze diversification in investments. (MA.912.F.4.10)
Benchmark: 11. Purchase stock with a set amount of money and follow the process through gains, losses, and selling. (MA.912.F.4.11)
Benchmark: 12. Compare and contrast income from purchase of common stock, preferred stock, and bonds. (MA.912.F.4.12)
Benchmark: 13. Given current exchange rates be able to convert from one form of currency to another. (MA.912.F.4.13)
Benchmark: 14. Use data to compare historical rates of return on investments with investment claims to make informed decisions and identify potential fraud. (MA.912.F.4.14)

Standard 5: Economic Concepts - Economic Concepts (MA.912.F.5)

Benchmark: 1. Demonstrate how price and quantity demanded relate, how price and quantity supplied relate, and how price changes or price controls affect distribution and allocation in the economy. (MA.912.F.5.1)
Benchmark: 2. Use basic terms and indicators associated with levels of economic performance and the state of the economy. (MA.912.F.5.2)

Geometry

Standard 1: Points, Lines, Angles, and Planes - Students understand geometric concepts, applications, and their representations with coordinate systems. They find lengths and midpoints of line segments, slopes, parallel and perpendicular lines, and equations of lines. Using a compass and straightedge, patty paper, a drawing program or other techniques, students also construct lines and angles, explaining and justifying the processes they use. (MA.912.G.1)

Benchmark: 1. * Find the lengths and midpoints of line segments in two-dimensional coordinate systems. (MA.912.G.1.1)
Benchmark: 2. Construct congruent segments and angles, angle bisectors, and parallel and perpendicular lines using a straight edge and compass or a drawing program, explaining and justifying the process used. (MA.912.G.1.2)
Benchmark: 3. * Identify and use the relationships between special pairs of angles formed by parallel lines and transversals. (MA.912.G.1.3)
Benchmark: 4. * Use coordinate geometry to find slopes, parallel lines, perpendicular lines, and equations of lines. (MA.912.G.1.4)

Standard 2: Polygons - Students identify and describe polygons (triangles, quadrilaterals, pentagons, hexagons, etc.), using terms such as regular, convex, and concave. They find measures of angles, sides, perimeters, and areas of polygons, justifying their methods. They apply transformations to polygons. They relate geometry to algebra by using coordinate geometry to determine transformations. Students use algebraic reasoning to determine congruence, similarity, and symmetry. Students create and verify tessellations of the plane using polygons. (MA.912.G.2)

Benchmark: 1. * Identify and describe convex, concave, regular, and irregular polygons. (MA.912.G.2.1)
Benchmark: 2. * Determine the measures of interior and exterior angles of polygons, justifying the method used. (MA.912.G.2.2)
Benchmark: 3. * Use properties of congruent and similar polygons to solve mathematical or real-world problems. (MA.912.G.2.3)
Benchmark: 4. * Apply transformations (translations, reflections, rotations, dilations, and scale factors) to polygons to determine congruence, similarity, and symmetry.0 Know that images formed by translations, reflections, and rotations are congruent to the original shape. Create and verify tessellations of the plane using polygons. (MA.912.G.2.4)
Benchmark: 5. * Explain the derivation and apply formulas for perimeter and area of polygons (triangles, quadrilaterals, pentagons, etc.). (MA.912.G.2.5)
Benchmark: 6. * Use coordinate geometry to prove properties of congruent, regular and similar polygons, and to perform transformations in the plane. (MA.912.G.2.6)
Benchmark: 7. * Determine how changes in dimensions affect the perimeter and area of common geometric figures. (MA.912.G.2.7)

Standard 3: Quadrilaterals - Students classify and understand relationships among quadrilaterals (rectangle, parallelogram, kite, etc.). They relate geometry to algebra by using coordinate geometry to determine regularity, congruence, and similarity. They use properties of congruent and similar quadrilaterals to solve problems involving lengths and areas, and prove theorems involving quadrilaterals. (MA.912.G.3)

Benchmark: 1. * Describe, classify, and compare relationships among quadrilaterals including the square, rectangle, rhombus, parallelogram, trapezoid, and kite. (MA.912.G.3.1)
Benchmark: 2. * Compare and contrast special quadrilaterals on the basis of their properties. (MA.912.G.3.2)
Benchmark: 3. * Use coordinate geometry to prove properties of congruent, regular and similar quadrilaterals. (MA.912.G.3.3)
Benchmark: 4. Prove theorems involving quadrilaterals. (MA.912.G.3.4)

Standard 4: Triangles - Students identify and describe various kinds of triangles (right, acute, scalene, isosceles, etc.). They define and construct altitudes, medians, and bisectors, and triangles congruent to given triangles. They prove that triangles are congruent or similar and use properties of these triangles to solve problems involving lengths and areas. They relate geometry to algebra by using coordinate geometry to determine regularity, congruence, and similarity. They understand and apply the inequality theorems of triangles. (MA.912.G.4)

Benchmark: 1. * Classify, construct, and describe triangles that are right, acute, obtuse, scalene, isosceles, equilateral, and equiangular. (MA.912.G.4.1)
Benchmark: 2. * Define, identify, and construct altitudes, medians, angle bisectors, perpendicular bisectors, orthocenter, centroid, incenter, and circumcenter. (MA.912.G.4.2)
Benchmark: 3. Construct triangles congruent to given triangles. (MA.912.G.4.3)
Benchmark: 4. * Use properties of congruent and similar triangles to solve problems involving lengths and areas. (MA.912.G.4.4)
Benchmark: 5. * Apply theorems involving segments divided proportionally. (MA.912.G.4.5)
Benchmark: 6. * Prove that triangles are congruent or similar and use the concept of corresponding parts of congruent triangles. (MA.912.G.4.6)
Benchmark: 7. * Apply the inequality theorems: triangle inequality, inequality in one triangle, and the Hinge Theorem. (MA.912.G.4.7)
Benchmark: 8. Use coordinate geometry to prove properties of congruent, regular, and similar triangles. (MA.912.G.4.8)

Standard 5: Right Triangles - Students apply the Pythagorean Theorem to solving problems, including those involving the altitudes of right triangles and triangles with special angle relationships. Students use special right triangles to solve problems using the properties of triangles. (MA.912.G.5)

Benchmark: 1. * Prove and apply the Pythagorean Theorem and its converse. (MA.912.G.5.1)
Benchmark: 2. State and apply the relationships that exist when the altitude is drawn to the hypotenuse of a right triangle. (MA.912.G.5.2)
Benchmark: 3. * Use special right triangles (30 (MA.912.G.5.3)
Benchmark: 4. Solve real-world problems involving right triangles. (MA.912.G.5.4)

Standard 6: Circles - Students define and understand ideas related to circles (radius, tangent, chord, etc.). They perform constructions and prove theorems related to circles. They find measures of arcs and angles related to them, as well as measures of circumference and area. They relate geometry to algebra by finding the equation of a circle in the coordinate plane. (MA.912.G.6)

Benchmark: 1. Determine the center of a given circle. Given three points not on a line, construct the circle that passes through them. Construct tangents to circles. Circumscribe and inscribe circles about and within triangles and regular polygons. (MA.912.G.6.1)
Benchmark: 2. * Define and identify: circumference, radius, diameter, arc, arc length, chord, secant, tangent and concentric circles. (MA.912.G.6.2)
Benchmark: 3. Prove theorems related to circles, including related angles, chords, tangents, and secants. (MA.912.G.6.3)
Benchmark: 4. * Determine and use measures of arcs and related angles (central, inscribed, and intersections of secants and tangents). (MA.912.G.6.4)
Benchmark: 5. * Solve real-world problems using measures of circumference, arc length, and areas of circles and sectors. (MA.912.G.6.5)
Benchmark: 6. * Given the center and the radius, find the equation of a circle in the coordinate plane or given the equation of a circle in center-radius form, state the center and the radius of the circle. (MA.912.G.6.6)
Benchmark: 7. Given the equation of a circle in center-radius form or given the center and the radius of a circle, sketch the graph of the circle. (MA.912.G.6.7)

Standard 7: Polyhedra and Other Solids - Students describe and make regular and nonregular polyhedra (cube, pyramid, tetrahedron, octahedron, etc.). They explore relationships among the faces, edges, and vertices of polyhedra. They describe sets of points on spheres, using terms such as great circle. They describe symmetries of solids and understand the properties of congruent and similar solids. (MA.912.G.7)

Benchmark: 1. * Describe and make regular, non-regular, and oblique polyhedra and sketch the net for a given polyhedron and vice versa. (MA.912.G.7.1)
Benchmark: 2. Describe the relationships between the faces, edges, and vertices of polyhedra. (MA.912.G.7.2)
Benchmark: 3. Identify, sketch, and determine areas and/or perimeters of cross sections of three-dimensional solids. (MA.912.G.7.3)
Benchmark: 4. * Identify chords, tangents, radii, and great circles of spheres. (MA.912.G.7.4)
Benchmark: 5. * Explain and use formulas for lateral area, surface area, and volume of three-dimensional solids. (MA.912.G.7.5)
Benchmark: 6. * Identify and use properties of congruent and similar three-dimensional solids. (MA.912.G.7.6)
Benchmark: 7. * Determine how changes in dimensions affect the surface area and volume of common three-dimensional geometric solids. (MA.912.G.7.7)

Standard 8: Mathematical Reasoning and Problem Solving - In a general sense, mathematics is problem solving. In all mathematics, students use problem-solving skills: they choose how to approach a problem, they explain their reasoning, and they check their results. At this level, students apply these skills to making conjectures, using axioms and theorems, constructing logical

arguments, and writing geometric proofs. They also learn about inductive and deductive reasoning and how to use counterexamples to show that a general statement is false. (MA.912.G.8)

Benchmark: 1. Analyze the structure of Euclidean geometry as an axiomatic system. Distinguish between undefined terms, definitions, postulates and theorems. (MA.912.G.8.1)
Benchmark: 2. Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guess-and-check, solving a simpler problem, writing an equation, and working backwards. (MA.912.G.8.2)
Benchmark: 3. * Determine whether a solution is reasonable in the context of the original situation. (MA.912.G.8.3)
Benchmark: 4. * Make conjectures with justifications about geometric ideas. Distinguish between information that supports a conjecture and the proof of a conjecture. (MA.912.G.8.4)
Benchmark: 5. Write geometric proofs, including proofs by contradiction and proofs involving coordinate geometry. Use and compare a variety of ways to present deductive proofs, such as flow charts, paragraphs, two-column, and indirect proofs. (MA.912.G.8.5)
Benchmark: 6. Perform basic constructions using straightedge and compass, and/or drawing programs describing and justifying the procedures used. Distinguish between sketching, constructing and drawing geometric figures. (MA.912.G.8.6)

Probability

Standard 1: Counting Principals - Students understand the counting principle, permutations, and combinations and use them to solve problems. (MA.912.P.1)

Benchmark: 1. * Use counting principles, including the addition and the multiplication principles, to determine size of finite sample spaces and probabilities of events in those spaces. (MA.912.P.1.1)
Benchmark: 2. Use formulas for permutations and combinations to count outcomes and determine probabilities of events. (MA.912.P.1.2)

Standard 2: Determine Probabilities - Students develop rules for finding probabilities of combined and complementary events. They understand and use conditional probability and the related Bayes (MA.912.P.2)

Benchmark: 1. * Determine probabilities of complementary events, and calculate odds for and against the occurrence of events. (MA.912.P.2.1)
Benchmark: 2. * Determine probabilities of independent events. (MA.912.P.2.2)
Benchmark: 3. Understand and use the concept of conditional probability, including: understanding how conditioning affects the probability of events; finding conditional probabilities from a two-way frequency table. (MA.912.P.2.3)

Standard 3: Probability Distributions - Students investigate probability distributions and calculate and interpret their means and variances. They use and apply the normal distribution, including using the central limit theorem. (MA.912.P.3)

Benchmark: 1. Determine probabilities of events from distributions, including:

- discrete uniform (all outcomes in a finite set equally likely)
- binomial
- normal
- exponential (MA.912.P.3.1)

Benchmark: 2. Determine the mean and variance of distributions, including:

- discrete uniform (all outcomes in a finite set equally likely)
- binomial
- normal
- exponential (MA.912.P.3.2)

Benchmark: 3. Apply the properties of the normal distribution. (MA.912.P.3.3)
Benchmark: 4. Apply the Central Limit Theorem to determine the probability that a sample mean will be in a certain interval. (MA.912.P.3.4)

Statistics

Standard 1: Formulating Questions - Students learn to define appropriate questions for research, and to pose questions in a form that can be answered by collecting and analyzing data. (MA.912.S.1)

Benchmark: 1. Formulate an appropriate research question to be answered by collecting data or performing an experiment. (MA.912.S.1.1)

Benchmark: 2. Determine appropriate and consistent standards of measurement for the data to be collected in a survey or experiment. (MA.912.S.1.2)

Standard 2: Data Collection - Students learn key methods for collecting data and basic sampling principles. (MA.912.S.2)

Benchmark: 1. Compare the difference between surveys, experiments, and observational studies, and what types of questions can and cannot be answered by a particular design. (MA.912.S.2.1)
Benchmark: 2. Apply the definition of random sample and basic types of sampling, including representative samples, stratified samples, censuses. (MA.912.S.2.2)
Benchmark: 3. * Identify sources of bias, including sampling and non-sampling errors. (MA.912.S.2.3)

Standard 3: Summarizing Data (Descriptive Statistics) - Students learn to work with summary measures of sets of data, including measures of the center, spread, and strength of relationship between variables. Students learn to distinguish between different types of data and to select the appropriate visual form to present different types of data. (MA.912.S.3)

Benchmark: 1. * Read and interpret data presented in various formats. Determine whether data is presented in appropriate format, and identify possible corrections. Formats to include:

- bar graphs
- line graphs
- stem and leaf plots
- circle graphs
- histograms
- box and whiskers plots
- scatter plots
- cumulative frequency (ogive) graphs (MA.912.S.3.1)

Benchmark: 2. * Collect, organize, and analyze data sets, determine the best format for the data and present visual summaries from the following:

- bar graphs
- line graphs
- stem and leaf plots
- circle graphs
- histograms
- box and whiskers plots
- scatter plots
- cumulative frequency (ogive) graphs (MA.912.S.3.2)

Benchmark: 3. * Calculate and interpret measures of the center of a set of data, including mean, median, and weighted mean, and use these measures to make comparisons among sets of data. (MA.912.S.3.3)
Benchmark: 4. Calculate and interpret measures of variance and standard deviation. Use these measures to make comparisons among sets of data. (MA.912.S.3.4)
Benchmark: 5. * Calculate and interpret the range and quartiles of a set of data. (MA.912.S.3.5)
Benchmark: 6. Use empirical rules (e.g. 68-95-99.7 rule) to estimate spread of distributions and to make comparisons among sets of data. (MA.912.S.3.6)
Benchmark: 7. Calculate the correlation coefficient of a set of paired data, and interpret the coefficient as a measure of the strength and direction of the relationship between the variables. (MA.912.S.3.7)
Benchmark: 8. Determine whether a data distribution is symmetric or skewed based on an appropriate graphical presentation of the data. (MA.912.S.3.8)
Benchmark: 9. Identify outliers in a set of data based on an appropriate graphical presentation of the data, and describe the effect of outliers on the mean, median, and range of the data. (MA.912.S.3.9)

Standard 4: Analyzing Data - Students learn to use simulations of standard sampling distributions to determine confidence levels and margins of error. They develop measures of association between two numerical or categorical variables. They can use technological tools to find equations of regression lines and correlation coefficients. (MA.912.S.4)

Benchmark: 1. Explain and interpret the concepts of confidence level and (MA.912.S.4.1)
Benchmark: 2. Use a simulation to approximate sampling distributions for the mean, using repeated sampling simulations from a given population. (MA.912.S.4.2)
Benchmark: 3. Apply the Central Limit Theorem to solve problems. (MA.912.S.4.3)
Benchmark: 4. Approximate confidence intervals for means using simulations of the distribution of the sample mean. (MA.912.S.4.4)
Benchmark: 5. Find the equation of the least squares regression line for a set of data. (MA.912.S.4.5)

Standard 5: Interpreting Results - Students gather data and determine confidence intervals to make inferences about means and use hypothesis tests to make decisions. They learn to use data to approximate p-values and to determine whether correlations between variables are significant. (MA.912.S.5)

Benchmark: 1. Analyze the relationship between confidence level, margin of error and sample size. (MA.912.S.5.1)
Benchmark: 2. Apply the general principles of hypothesis testing. (MA.912.S.5.2)
Benchmark: 3. Explain and identify the following: null hypothesis, alternative hypotheses, Type I error, and Type II error. (MA.912.S.5.3)
Benchmark: 4. Explain the meaning of p-value and its role in hypothesis testing. (MA.912.S.5.4)
Benchmark: 5. Perform hypothesis tests of means and proportions for large samples, using simulations to determine whether a sample mean (proportion) has a low likelihood of occurring. (MA.912.S.5.5)
Benchmark: 6. Interpret the results of hypothesis tests of means and proportions, and make decisions based on p-values of test. (MA.912.S.5.6)
Benchmark: 7. Use simulations to approximate the p-value of a correlation coefficient, and use the results to determine whether the correlation between two variables is significant. (MA.912.S.5.7)
Benchmark: 8. Use a regression line equation to make predictions. (MA.912.S.5.8)
Benchmark: 9. Interpret the coefficient of determination, r2, for a least-squares regression. (MA.912.S.5.9)

Trigonometry

Standard 1: Trigonometric Functions - Students extend the definitions of the trigonometric functions beyond right triangles using the unit circle and they measure angles in radians as well as degrees. They draw and analyze graphs of trigonometric functions (including finding period, amplitude, and phase shift) and use them to solve word problems. They define and graph inverse trigonometric functions and determine values of both trigonometric and inverse trigonometric functions. (MA.912.T.1)

Benchmark: 1. Convert between degree and radian measures. (MA.912.T.1.1)
Benchmark: 2. Define and determine sine and cosine using the unit circle. (MA.912.T.1.2)
Benchmark: 3. State and use exact values of trigonometric functions for special angles, i.e. multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$ (degree and radian measures) (MA.912.T.1.3)
Benchmark: 4. Find approximate values of trigonometric and inverse trigonometric functions using appropriate technology. (MA.912.T.1.4)
Benchmark: 5. Make connections between right triangle ratios, trigonometric functions, and circular functions. (MA.912.T.1.5)
Benchmark: 6. Define and graph trigonometric functions using domain, range, intercepts, period, amplitude, phase shift, vertical shift, and asymptotes with and without the use of graphing technology. (MA.912.T.1.6)
Benchmark: 7. Define and graph inverse trigonometric relations and functions. (MA.912.T.1.7)
Benchmark: 8. Solve real-world problems involving applications of trigonometric functions using graphing technology when appropriate. (MA.912.T.1.8)

Standard 2: Trigonometry in Triangles - Students understand how the trigonometric functions relate to right triangles and solve word problems involving right and oblique triangles. They understand and apply the laws of sines and cosines. They use trigonometry to find the area of triangles. (MA.912.T.2)

Benchmark: 1. * Define and use the trigonometric ratios (sine, cosine, tangent, cotangent, secant, and cosecant) in terms of angles of right triangles. (MA.912.T.2.1)
Benchmark: 2. * Solve real-world problems involving right triangles using technology when appropriate. (MA.912.T.2.2)
Benchmark: 3. Apply the laws of sines and cosines to solve real-world problems using technology. (MA.912.T.2.3)
Benchmark: 4. Use the area of triangles given two sides and an angle or three sides to solve real-world problems. (MA.912.T.2.4)

Standard 3: Trigonometric Identities and Equations - Students know basic trigonometric identities derived from definitions and use them to prove other identities. They use the sum, difference, double-angle, and half-angle formulas. They solve trigonometric equations and word problems using trigonometry. (MA.912.T.3)

Benchmark: 1. Verify the basic Pythagorean identities, e.g., sin2x + cos2x = 1, and show they are equivalent to the Pythagorean Theorem. (MA.912.T.3.1)
Benchmark: 2. Use basic trigonometric identities to verify other identities and simplify expressions. (MA.912.T.3.2)
Benchmark: 3. Use the sum and difference, half-angle and double-angle formulas for sine, cosine, and tangent, when formulas are provided. (MA.912.T.3.3)
Benchmark: 4. Solve trigonometric equations and real-world problems involving applications of trigonometric equations using technology when appropriate. (MA.912.T.3.4)

Standard 4: Polar Coordinates and Trigonometric Form of Complex Numbers - Students define, use polar coordinates, and relate them to Cartesian coordinates. They translate equations in terms of Cartesian coordinates into polar coordinates and graph the resulting equations in the polar coordinate plane. They convert complex numbers from standard to trigonometric form, and vice-versa. They multiply complex numbers in trigonometric form and use De Moivre (MA.912.T.4)

Benchmark: 1. Define polar coordinates and relate polar coordinates to Cartesian coordinates with and without the use of technology. (MA.912.T.4.1)
Benchmark: 2. Represent equations given in rectangular coordinates in terms of polar coordinates. (MA.912.T.4.2)
Benchmark: 3. Graph equations in the polar coordinate plane with and without the use of graphing technology. (MA.912.T.4.3)
Benchmark: 4. Define the trigonometric form of complex numbers, convert complex numbers to trigonometric form, and multiply complex numbers in trigonometric form. (MA.912.T.4.4)
Benchmark: 5. Apply DeMoivre (MA.912.T.4.5)

Standard 5: Mathematical Reasoning and Problem Solving - Students use a variety of strategies to solve problems. They develop and evaluate mathematical arguments and proofs. (MA.912.T.5)

Benchmark: 1. Use a variety of problem-solving strategies, such as drawing a diagram, guess-and-check, solving a simpler problem, examining simpler problems, and working backwards, and using technology when appropriate. (MA.912.T.5.1)
Benchmark: 2. Decide whether a solution is reasonable in the context of the original situation. (MA.912.T.5.2)
Benchmark: 3. Determine whether a given trigonometric statement is always, sometimes, or never true. Use the properties of the real numbers, order of operations, and trigonometric identities to justify the steps involved in verifying identities and solving equations. (MA.912.T.5.3)

NGSSS: (Standard Numbers/Standards/Taxonomy Levels)

Next Generation Sunshine State Standards (Florida)

http://tools.fcit.usf.edu/sss/

Visual Art 9-12

Big Ideas: **Critical Thinking and Reflection (C),**

Skills, Techniques, and Processes (S),

Organizational Structure (O),

Historical and Global Connections (H),

Innovation, Technology, and the Future (F)

Enduring Understandings:

Big Idea: CRITICAL THINKING AND REFLECTION

Enduring Understanding 1: Cognition and reflection are required to appreciate, interpret, and create with artistic intent. (VA.912.C.1)

Benchmark: 1. Integrate curiosity, range of interests, attentiveness, complexity, and artistic intention in the art-making process to demonstrate self-expression. (VA.912.C.1.1)
Benchmark: 2. Use critical-thinking skills for various contexts to develop, refine, and reflect on an artistic theme. (VA.912.C.1.2)
Benchmark: 3. Evaluate the technical skill, aesthetic appeal, and/or social implication of artistic exemplars to formulate criteria for assessing personal work. (VA.912.C.1.3)
Benchmark: 4. Apply art knowledge and contextual information to analyze how content and ideas are used in works of art. (VA.912.C.1.4)

Benchmark: 5. Analyze how visual information is developed in specific media to create a recorded visual image. (VA.912.C.1.5)
Benchmark: 6. Identify rationale for aesthetic choices in recording visual media. (VA.912.C.1.6)
Benchmark: 7. Analyze challenges and identify solutions for three-dimensional structural problems. (VA.912.C.1.7)
Benchmark: 8. Explain the development of meaning and procedural choices throughout the creative process to defend artistic intention. (VA.912.C.1.8)

Enduring Understanding 2: Assessing our own and others' artistic work, using critical-thinking, problem-solving, and decision-making skills, is central to artistic growth. (VA.912.C.2)

Benchmark: 1. Examine and revise artwork throughout the art-making process to refine work and achieve artistic objective. (VA.912.C.2.1)
Benchmark: 2. Assess the works of others, using established or derived criteria, to support conclusions and judgments about artistic progress. (VA.912.C.2.2)
Benchmark: 3. Process and apply constructive criticism as formative assessment for continued growth in art-making skills.

(VA.912.C.2.3)
Benchmark: 4. Classify artworks, using accurate art vocabulary and knowledge of art history to identify and categorize movements, styles, techniques, and materials. (VA.912.C.2.4)
Benchmark: 5. Develop and use criteria to select works for a portfolio and defend one's artistic choices with a written, oral, and/or recorded analysis. (VA.912.C.2.5)
Benchmark: 6. Investigate the process of developing a coherent, focused concept in a body of work comprised of multiple artworks. (VA.912.C.2.6)
Benchmark: 7. Assess the challenges and outcomes associated with the media used in a variety of one's own works. (VA.912.C.2.7)
Benchmark: 8. Compare artwork, architecture, designs, and/or models to understand how technical and utilitarian components impact aesthetic qualities. (VA.912.C.2.8)

Enduring Understanding 3: The processes of critiquing works of art lead to development of critical-thinking skills transferable to other contexts. (VA.912.C.3)

Benchmark: 1. Use descriptive terms and varied approaches in art analysis to explain the meaning or purpose of an artwork. (VA.912.C.3.1)
Benchmark: 2. Develop and apply criteria to determine how aesthetic works are aligned with a personal definition of "art." (VA.912.C.3.2)
Benchmark: 3. Examine relationships among social, historical, literary, and/or other references to explain how they are assimilated into artworks. (VA.912.C.3.3)
Benchmark: 4. Use analytical skills to examine issues in non-visual art contexts. (VA.912.C.3.4)
Benchmark: 5. Make connections between timelines in other content areas and timelines in the visual arts. (VA.912.C.3.5)
Benchmark: 6. Discuss how the aesthetics of artwork and utilitarian objects have changed over time. (VA.912.C.3.6)

Big Idea: SKILLS, TECHNIQUES, AND PROCESSES

Enduring Understanding 1: The arts are inherently experiential and actively engage learners in the processes of creating, interpreting, and responding to art. (VA.912.S.1)

Benchmark: 1. Use innovative means and perceptual understanding to communicate through varied content, media, and art techniques. (VA.912.S.1.1)
Benchmark: 2. Investigate the use of technology and other resources to inspire art-making decisions. (VA.912.S.1.2)
Benchmark: 3. Interpret and reflect on cultural and historical events to create art. (VA.912.S.1.3)
Benchmark: 4. Demonstrate effective and accurate use of art vocabulary throughout the art-making process. (VA.912.S.1.4)
Benchmark: 5. Compare the aesthetic impact of images created with different media to evaluate advantages or disadvantages within the art process. (VA.912.S.1.5)
Benchmark: 6. Describe processes and techniques used to record visual imagery. (VA.912.S.1.6)
Benchmark: 7. Manipulate lighting effects, using various media to create desired results. (VA.912.S.1.7)
Benchmark: 8. Use technology to simulate art-making processes and techniques. (VA.912.S.1.8)
Benchmark: 9. Use diverse media and techniques to create paintings that represent various genres and schools of painting. (VA.912.S.1.9)

Enduring Understanding 2: Development of skills, techniques, and processes in the arts strengthens our ability to remember, focus on, process, and sequence information. (VA.912.S.2)

Benchmark: 1. Demonstrate organizational skills to influence the sequential process when creating artwork. (VA.912.S.2.1)
Benchmark: 2. Focus on visual information and processes to complete the artistic concept. (VA.912.S.2.2)
Benchmark: 3. Demonstrate visual-thinking skills to process the challenges and execution of a creative endeavor. (VA.912.S.2.3)
Benchmark: 4. Use information resources to develop concepts representing diversity and effectiveness for using selected media and techniques in a sketchbook or journal. (VA.912.S.2.4)
Benchmark: 5. Demonstrate use of perceptual, observational, and compositional skills to produce representational, figurative, or abstract imagery. (VA.912.S.2.5)
Benchmark: 6. Incorporate skills, concepts, and media to create images from ideation to resolution. (VA.912.S.2.6)

Enduring Understanding 3: Through purposeful practice, artists learn to manage, master, and refine simple, then complex, skills and techniques. (VA.912.S.3)

Benchmark: 1. Manipulate materials, techniques, and processes through practice and perseverance to create a desired result in two- and/or three-dimensional artworks. (VA.912.S.3.1)
Benchmark: 2. Demonstrate a balance between spontaneity and purpose to produce complex works of art with conviction and disciplined craftsmanship. (VA.912.S.3.2)
Benchmark: 3. Review, discuss, and demonstrate the proper applications and safety procedures for hazardous chemicals and equipment during the art-making process. (VA.912.S.3.3)
Benchmark: 4. Demonstrate personal responsibility, ethics, and integrity, including respect for intellectual property, when accessing information and creating works of art. (VA.912.S.3.4)
Benchmark: 5. Create multiple works that demonstrate thorough exploration of subject matter and themes. (VA.912.S.3.5)
Benchmark: 6. Develop works with prominent personal vision revealed through mastery of art tasks and tools. (VA.912.S.3.6)

Benchmark: 7. Use and maintain tools and equipment to facilitate the creative process. (VA.912.S.3.7)
Benchmark: 8. Develop color-mixing skills and techniques through application of the principles of heat properties and color and light theory. (VA.912.S.3.8)
Benchmark: 9. Manipulate and embellish malleable or rigid materials to construct representational or abstract forms. (VA.912.S.3.9)
Benchmark: 10. Develop skill in sketching and mark-making to plan, execute, and construct two-dimensional images or three-dimensional models. (VA.912.S.3.10)
Benchmark: 11. Store and maintain equipment, materials, and artworks properly in the art studio to prevent damage and/or cross-contamination. (VA.912.S.3.11)
Benchmark: 12. Develop competence and dexterity, through practice, in the use of processes, tools, and techniques for various media. (VA.912.S.3.12)
Benchmark: 13. Create three-dimensional modeled and rendered objects in figurative and nonfigurative digital applications. (VA.912.S.3.13)

Big Idea: ORGANIZATIONAL STRUCTURE

Enduring Understanding 1: Understanding the organizational structure of an art form provides a foundation for appreciation of artistic works and respect for the creative process. (VA.912.O.1)

Benchmark: 1. Use the structural elements of art and the organizational principles of design in works of art to establish an interpretive and technical foundation for visual coherence. (VA.912.O.1.1)
Benchmark: 2. Use and defend the choice of creative and technical skills to produce artworks. (VA.912.O.1.2)
Benchmark: 3. Research and use the techniques and processes of various artists to create personal works. (VA.912.O.1.3)
Benchmark: 4. Compare and analyze traditional and digital media to learn how technology has altered opportunities for innovative responses and results. (VA.912.O.1.4)
Benchmark: 5. Investigate the use of space, scale, and environmental features of a structure to create three-dimensional form or the illusion of depth and form. (VA.912.O.1.5)

Enduring Understanding 2: The structural rules and conventions of an art form serve as both a foundation and departure point for creativity. (VA.912.O.2)

Benchmark: 1. Construct new meaning through shared language, ideation, expressive content, and unity in the creative process. (VA.912.O.2.1)
Benchmark: 2. Solve aesthetic problems, through convergent and divergent thinking, to gain new perspectives. (VA.912.O.2.2)
Benchmark: 3. Investigate an idea in a coherent and focused manner to provide context in the visual arts. (VA.912.O.2.3)
Benchmark: 4. Concentrate on a particular style, theme, concept, or personal opinion to develop artwork for a portfolio, display, or exhibition. (VA.912.O.2.4)

Enduring Understanding 3: Every art form uses its own unique language, verbal and non-verbal, to document and communicate with the world. (VA.912.O.3)

Benchmark: 1. Create works of art that include symbolism, personal experiences, or philosophical view to communicate with an audience. (VA.912.O.3.1)
Benchmark: 2. Create a series of artworks to inform viewers about personal opinions and/or current issues. (VA.912.O.3.2)

Big Idea: HISTORICAL AND GLOBAL CONNECTIONS

Enduring Understanding 1: Through study in the arts, we learn about and honor others and the worlds in which they live(d). (VA.912.H.1)

Benchmark: 1. Analyze the impact of social, ecological, economic, religious, and/or political issues on the function or meaning of the artwork. (VA.912.H.1.1)
Benchmark: 2. Analyze the various functions of audience etiquette to formulate guidelines for conduct in different art venues. (VA.912.H.1.2)
Benchmark: 3. Examine the significance placed on art forms over time by various groups or cultures compared to current views on aesthetics. (VA.912.H.1.3)
Benchmark: 4. Apply background knowledge and personal interpretation to discuss cross-cultural connections among various artworks and the individuals, groups, cultures, events, and/or traditions they reflect. (VA.912.H.1.4)
Benchmark: 5. Investigate the use of technology and media design to reflect creative trends in visual culture. (VA.912.H.1.5)
Benchmark: 6. Create a timeline for the development of artists' materials to show multiple influences on the use of art media. (VA.912.H.1.6)
Benchmark: 7. Research and report technological developments to identify influences on society. (VA.912.H.1.7)
Benchmark: 8. Analyze and compare works in context, considering economic, social, cultural, and political issues, to define the significance and purpose of art. (VA.912.H.1.8)
Benchmark: 9. Describe the significance of major artists, architects, or masterworks to understand their historical influences. (VA.912.H.1.9)

Benchmark: 10. Describe and analyze the characteristics of a culture and its people to create personal art reflecting daily life and/or the specified environment. (VA.912.H.1.10)

Enduring Understanding 2: The arts reflect and document cultural trends and historical events, and help explain how new directions in the arts have emerged. (VA.912.H.2)

Benchmark: 1. Identify transitions in art media, technique, and focus to explain how technology has changed art throughout history. (VA.912.H.2.1)
Benchmark: 2. Analyze the capacity of the visual arts to fulfill aesthetic needs through artwork and utilitarian objects. (VA.912.H.2.2)
Benchmark: 3. Analyze historical or cultural references in commemorative works of art to identify the significance of the event or person portrayed. (VA.912.H.2.3)
Benchmark: 4. Research the history of art in public places to examine the significance of the artwork and its legacy for the future. (VA.912.H.2.4)
Benchmark: 5. Analyze artwork from a variety of cultures and times to compare the function, significance, and connection to other cultures or times. (VA.912.H.2.5)
Benchmark: 6. Analyze artistic trends to explain the rationale for creating personal adornment, visual culture, and/or design. (VA.912.H.2.6)

Enduring Understanding 3: Connections among the arts and other disciplines strengthen learning and the ability to transfer knowledge and skills to and from other fields. (VA.912.H.3)

Benchmark: 1. Synthesize knowledge and skills learned from non-art content areas to support the processes of creation, interpretation, and analysis. (VA.912.H.3.1)
Benchmark: 2. Apply the critical-thinking and problem-solving skills used in art to develop creative solutions for real-life issues. (VA.912.H.3.2)
Benchmark: 3. Use materials, ideas, and/or equipment related to other content areas to generate ideas and processes for the creation of works of art. (VA.912.H.3.3)

Big Idea: INNOVATION, TECHNOLOGY, AND THE FUTURE

Enduring Understanding 1: Creating, interpreting, and responding in the arts stimulate the imagination and encourage innovation and creative risk-taking. (VA.912.F.1)

Benchmark: 1. Use divergent thinking, abstract reasoning, and various processes to demonstrate imaginative or innovative solutions for art problems. (VA.912.F.1.1)
Benchmark: 2. Manipulate or synthesize established techniques as a foundation for individual style initiatives in two-, three-, and/or four-dimensional applications. (VA.912.F.1.2)
Benchmark: 3. Demonstrate flexibility and adaptability throughout the innovation process to focus and re-focus on an idea, deliberately delaying closure to promote creative risk-taking. (VA.912.F.1.3)
Benchmark: 4. Use technological tools to create art with varying effects and outcomes. (VA.912.F.1.4)
Benchmark: 5. Create a digital or time-based presentation to analyze and compare artists, artworks, and concepts in historical context. (VA.912.F.1.5)

Enduring Understanding 2: Careers in and related to the arts significantly and positively impact local and global economies. (VA.912.F.2)

Benchmark: 1. Examine career opportunities in the visual arts to determine requisite skills, qualifications, supply-and-demand, market location, and potential earnings. (VA.912.F.2.1)
Benchmark: 2. Examine a broad spectrum of art-related careers to identify potential employment opportunities that involve construction, management, and/or sale of aesthetic or utilitarian objects. (VA.912.F.2.2)
Benchmark: 3. Analyze the potential economic impact of arts entities to revitalize a community or region. (VA.912.F.2.3)
Benchmark: 4. Research ideas to plan, develop, and market art-related goods, artworks, or services that influence consumer beliefs and behaviors. (VA.912.F.2.4)
Benchmark: 5. Develop a personal artist statement, résumé, presentation, or digital portfolio to interview for an art-related position or exhibition. (VA.912.F.2.5)
Benchmark: 6. Research and discuss the potential of the visual arts to improve aesthetic living. (VA.912.F.2.6)
Benchmark: 7. Evaluate the effects of creating works of art for sale or donation to support local organizations for social or economic causes. (VA.912.F.2.7)
Benchmark: 8. Describe community resources to preserve, restore, exhibit, and view works of art. (VA.912.F.2.8)

Enduring Understanding 3: The 21st-century skills necessary for success as citizens, workers, and leaders in a global economy are embedded in the study of the arts. (VA.912.F.3)

Benchmark: 1. Use technology applications and art skills to promote social and cultural awareness regarding community initiatives and/or concerns. (VA.912.F.3.1)

Benchmark: 2. Examine the rationale for using procedural, analytical, and divergent thinking to achieve visual literacy. (VA.912.F.3.2)
Benchmark: 3. Discuss how the arts help students develop self-reliance and promote collaboration to strengthen leadership capabilities as priorities change. (VA.912.F.3.3)
Benchmark: 4. Follow directions and use effective time-management skills to complete the art-making process and show development of 21st-century skills. (VA.912.F.3.4)
Benchmark: 5. Use appropriately cited sources to document research and present information on visual culture. (VA.912.F.3.5)
Benchmark: 6. Identify ethical ways to use appropriation in personal works of art. (VA.912.F.3.6)
Benchmark: 7. Create a body of collaborative work to show artistic cohesiveness, team-building, respectful compromise, and time-management skills. (VA.912.F.3.7)
Benchmark: 8. Combine art and design skills with entrepreneurialism to provide community service and leverage strengths in accomplishing a common objective. (VA.912.F.3.8)
Benchmark: 9. Identify and apply collaborative procedures to coordinate a student or community art event. (VA.912.F.3.9)
Benchmark: 10. Apply rules of convention to create purposeful design. (VA.912.F.3.10)
Benchmark: 11. Demonstrate proficiency in creating individual and sequential images, animation, or media in motion with sound to solve visual problems. (VA.912.F.3.11)
Benchmark: 12. Use digital equipment and peripheral devices to record, create, present, and/or share accurate visual images with others. (VA.912.F.3.12)

Observations and Notes:

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